Sovereign Risk in International Bond Markets

and Nonconvergence

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Abstract

The paper analyzes the consequences of joining markets of government discount bonds between identical economies when, in each country, there exists a positive probability of the government to default. In autarky such economies of overlapping generations of consumers with capital accumulation converge to a unique positive steady state under certain conditions. When two identical economies of this type open their markets for bonds to the consumers of the other country, diversification in portfolio demand by consumers leads to the existence of a world equilibrium with a uniform bond price in every period. Under such circumstances a symmetric stationary world equilibrium with lower risk per country exists, supported by the same level of capital and income as under autarky. However, due to the interaction of dynamic spill over effects between the bond market and the domestic markets for capital investment, the symmetric steady state may become unstable and stable asymmetric steady states appear, implying symmetry breaking in the sense of Matsuyama (2004). The paper identifies a set of assumptions on consumer and production characteristics together with a range of values of the government's default parameters, such that instability occurs and asymmetric steady states become locally stable.

Keywords: capital accumulation, sovereign risk, international bond markets, non convergence, symmetry-breaking.

JEL classification: C62, D91, E20, E44, F36, F41, F43, G11, G12, G15, O11, O41

1 Introduction

The debate on the consequences of a liberalization of asset markets internationally has been carried out primarily under the premise that more trading options for investors between countries also increase real investment opportunities in each country, supporting or reinforcing an innate tendency to equalize incomes across countries. Therefore, many economists regard an asset market liberalization between countries not only as a device for additional risk sharing between countries, but also as an essential vehicle for convergence of per capita incomes across countries.

According to traditional views, liberalized asset markets facilitate the flow of financial resources from industrialized countries with abundant capital to developing countries where capital is scarce. It is usually argued that this flow reduces the cost of capital and increases capital investment in developing countries. In other words, if the flow of funds leads to more capital formation in the less developed country, the latter will grow faster inducing convergence of incomes, as argued for example by Fischer (1998) and Summers (2000). Some economists have expressed skepticism that such consequences are universal arguing that the liberalization of financial markets might promote a flow of funds from capital-scarce to capital-abundant countries as well. If present, these would affect the capital investment in poor countries adversely and could cause a serious impediment to income growth, inducing divergence rather than convergence of incomes across countries under liberalized asset markets. In such cases, a liberalization at times of uneven distributions of capital and income across countries might result in stationary states dividing the world into poor and rich countries permanently, when standard allocative economic arguments would predict a symmetric outcome. Matsuyama (2004, 2005) has introduced the term *symmetry breaking* for such a situation.

The recent literature identifies different structures in economic models why capital account liberalization may promote a flow of financial capital from capital-scarce to capital-abundant countries causing symmetry breaking. Imperfections in the credit market can be one such reason, leading to a reverse flow of capital from capital-scarce to capital-abundant countries as a result of capital account liberalization, as shown for example by Boyd & Smith (1997), Matsuyama (2004), and Kikuchi & Stachurski (2009). Another cause may be consequences of agglomeration in each country as an outcome of increasing returns to scale (see Krugman & Venables (1995) and Matsuyama (1996)). In such a case, the domestic rate of return on capital increases as more capital is allocated to that country. As a result, structurally identical economies can find themselves in quite dissimilar situations after liberalization of capital accounts, because higher returns not only attract more capital, but also more capital implies higher returns, due to agglomeration. In both cases the effects are caused by specific non convexities existing in each economy in an otherwise fully convex competitive world. Thus, both explanations support in principle the traditional view that more liberalization induces a beneficial effect to developing

countries and induces further convergence if these non convexities were not present. A third cause has been shown when there is uncertainty of the return of a private financial asset (shares or equity) arising from random dividends in production (see Böhm, Kikuchi & Vachadze (2007), Kikuchi (2008), and Böhm & Vachadze (2009)).

In all three cases, the free flow of financial resources after liberalization can cause even small existing differences across countries to magnify. As a result, the world economy may inevitably become divided endogenously into rich and poor countries. Financial capital will flow from poor to rich countries, despite the fact that the return on real capital is higher in developing countries. Rich countries are characterized by high income, high investment, and low credit market imperfection or low agglomeration, while poor countries suffer from low income, low investment, and high credit market imperfections or high agglomeration .

The present paper identifies the possibility of default of two separate but identical governments issuing bonds as a structural reason for symmetry breaking. From the consumer point of view, bonds are treated like any other asset in the portfolio decision problem, serving as an additional but *risky* investment opportunity to real capital. This makes the portfolio problem of consumers similar to the private asset case, but the dynamic structure under perfect foresight with private assets is structurally different and more involved than with bonds, since the former requires the solution of a functional equation. Moreover, the impact of the risk of default on the dynamics of capital accumulation is quite different from the three cases studied before. Under autarky such economies of overlapping generations of consumers, with a government and a bond market converge to a unique positive steady state when relative risk aversion is increasing and concave and when substitutability in production is sufficiently high. When the bond markets of two economies are liberalized internationally, symmetry breaking may occur, i. e. the symmetric steady state of the world economy with perfect risk diversification becomes unstable and asymmetric stable steady states emerge after consumers from both countries trade government bonds internationally.

The paper is organized as follows. After introducing the model in Section 2, Section 3 analyzes each economy in autarky and shows the existence and global stability of a unique and interior steady state. Section 4 analyzes a world economy composed of two identical economies and shows that symmetry breaking occurs provided a general elasticity condition holds at the symmetric steady state. Thus, the endogenous formation of rich and poor countries can be an inevitable outcome of internationalizing bond markets. Section 5 summarizes the results and concludes.

2 The Model

Consider a world economy with an infinite-horizon in discrete time. Each economy is composed of a government, a consumption and a production sector. The production sector consist of a large number of infinitely lived firms producing a single final commodity. The technology of each firm has constant returns to scale, is identical and constant through time. At any time t, a standard production function $F : \mathbb{R}^2_+ \to \mathbb{R}$ determines total output as $Y_t = F(K_t, L_t)$, where $K_t \geq 0$ and $L_t \geq 0$ are aggregate supplies of physical capital and labor respectively. Let $k_t = K_t/L_t$ denote capital per worker and let $f : \mathbb{R}_+ \to \mathbb{R}_+$ denote the production function in intensity form. Hence, output per worker is $y_t = Y_t/L_t = F(K_t/L_t, 1) \equiv f(k_t)$. f is assumed to be twice continuously differentiable, strictly increasing, strictly concave, satisfying the Inada conditions. In addition, $f(0) = 0$ holds, implying that capital is essential in production.

Both factor markets are competitive so that factor rewards on physical capital and labor are equal to their respective marginal products. Thus, $r_t = r(k_t) \equiv f'(k_t)$ is the rental rate of capital and $w_t = w(k_t) \equiv f(k_t) - k_t f'(k_t)$ is the wage rate in a given period. The produced commodity can be either consumed or invested in physical capital, which becomes available in the next period. Capital depreciates fully within a period.

The sovereign governments in each country face a stream of public expenditures, which is exogenous, stochastic, and unproductive. These expenditures are financed by levying a proportional tax on labor income at the rate $0 \leq \tau \leq 1$ and by issuing one-period pure discount bonds in the amount of $b > 0$ with default option. One discount bond pays one unit of consumption in the subsequent period if the government does not default its obligation. It pays $\delta \in [0, 1)$ units of goods otherwise. The parameter δ measures the debt recovery rate, which is the fraction of debt repaid if the sovereign defaults on his obligation. The governments' no-default/default situations are exogenous and random and occurs independently with probabilities $p \in (0, 1)$ and $1 - p$ respectively.

The consumption sector of the economy is described by two period lived consumers. There are two generations alive in each period, called young and old, where each generation consists of a large number of homogeneous agents. There is no population growth. A typical young agent is endowed with one unit of labor, which he supplies inelastically to the labor market where he earns the wage income w_t. There is no consumption in the first period. After paying the wage tax, a young consumer chooses a portfolio consisting of real savings/investment (which becomes real capital in the next period) and portfolio of bonds. Consumers are risk averse and their preferences over the second period consumption is described by a utility function $u : \mathbb{R}_+ \to \mathbb{R}$, which is three times continuously differentiable, non-negative, strictly increasing, and strictly concave.

We consider two cases separately. First we consider the case when the bond markets operate only domestically and young consumers purchase only their own country's discount

bonds. In the second case the bond markets operate internationally and consumers in the two countries can buy bonds of both countries in an international bond market. In other words, consumers of each country can invest in domestic capital and can purchase bonds of both countries.

3   The Closed Economy

Consider first the case of autarky with a domestic bond market only. A young agent's budget constraint is $i_t + x_t q_t \leq (1 - \tau)w_t$, where i_t denotes the amount of physical capital investment and x_t is the number of discount bonds purchased at a price q_t. Young consumers cannot take a short position in either of the two asset markets, i. e. they can neither obtain credit to finance their demand for bonds nor are they allowed to sell bonds in order to finance their capital investment. Therefore, $i_t \geq 0$ and $x_t \geq 0$ for any t. Suppose $x_t \geq 0$ bonds are purchased and r_{t+1} is next period's rental rate of capital. Then second period consumption

$$c_{t+1} = \begin{cases} (1 - \tau)w_t r_{t+1} + x_t(\delta - q_t r_{t+1}) & \text{with prob} \quad 1 - p \\ \\ (1 - \tau)w_t r_{t+1} + x_t(1 - q_t r_{t+1}) & \text{with prob} \quad p \end{cases}$$

is random which can be written as

$$c_{t+1} = ((1 - \tau)w_t - q_t x_t)\, r_{t+1} + x_t \left(1 - (1 - \delta)d_{t+1}\right), \tag{1}$$

where $d_{t+1} \in \{0, 1\}$ denotes the government's default indicator taking the values zero (case of no-default) with probability p and one (case of default) with probability $1 - p$. Observe that the first term in (1) is equal to the expected return $i_t r_{t+1}$ on real investment. Therefore, given the law of capital accumulation $k_{t+1} = i_t = (1 - \tau)w_t - q_t x_t$, under perfect foresight one may write $i_t r_{t+1} = \rho(k_{t+1}) = k_{t+1} f'(k_{t+1})$ where $\rho(k) := kf'(k)$ denotes capital income. Therefore, under perfect foresight, one may define

$$\bar{c}_{t+1} := \rho(k_{t+1}) + x_t \quad \text{and} \quad \underline{c}_{t+1} := \rho(k_{t+1}) + \delta x_t. \tag{2}$$

A young agent's objective is to maximize expected utility of second period consumption

$$\max_{x_t \in \mathcal{B}(q_t, w_t)} pu(\bar{c}_{t+1}) + (1 - p)\, u(\underline{c}_{t+1}), \tag{3}$$

where $\mathcal{B}(q_t, w_t) = \{x_t | x_t \geq 0, q_t x_t \leq (1 - \tau)w_t\}$ is the current budget set and $\bar{c}_{t+1}$ and $\underline{c}_{t+1}$ are the consumption levels in cases of the no-default and the default case respectively. The first order condition for an interior solution is

$$pu'(\bar{c}_{t+1})\left(1 - q_t r_{t+1}\right) + (1 - p)u'(\underline{c}_{t+1})\left(\delta - q_t r_{t+1}\right) = 0. \tag{4}$$

Under perfect foresight and using the law of capital accumulation as described above, equation (4) implies the following inverse demand function for the bond

$$q_t = H(k_{t+1}, x_t) := \frac{h(k_{t+1}, x_t)}{r(k_{t+1})}, \tag{5}$$

where

$$h(k_{t+1}, x_t) \equiv \frac{pu'(\rho(k_{t+1}) + x_t) + (1-p)\delta u'(\rho(k_{t+1}) + \delta x_t)}{pu'(\rho(k_{t+1}) + x_t) + (1-p)u'(\rho(k_{t+1}) + \delta x_t)} \in (0,1) \tag{6}$$

is the risk adjusted second period payment on the bond. Observe that the inverse demand function does not contain the wage income w_t as an argument any more, as one would expect from such an analysis. This is due to the fact that w_t appears linearly in the law of capital accumulation (no consumption when young!) and a consequence of perfect foresight. Together these conditions imply that the role of the current net wage cancels out in the inverse demand for bonds.

The results of the paper are driven primarily by properties of the production function and of the utility function, which restrict the curvature of these two concave functions. The first one imposes a lower bound on the elasticity of substitution in production between capital and labor, which is tantamount to a condition on the monotonicity of the capital return. The second one restricts the absolute risk aversion of the consumer to be non increasing with an elasticity between minus one and zero.

Assumption 1 *The production function f is such that capital income $\rho(k) := kf'(k)$ is a strictly increasing function.*

In other words, with differentiability and strict concavity of f, the elasticity of f' must be greater than minus one, $\epsilon_{f'}(k) \in (-1, 0)$, for every k. Equivalently, this implies also that the elasticity of substitution between capital and labor $\sigma : \mathbb{R}_+ \to \mathbb{R}_+$ and the capital share in production $\alpha : \mathbb{R}_+ \to [0, 1]$, defined as

$$\sigma(k) := -\frac{f'(k)(f(k) - kf'(k))}{kf''(k)f(k)} \quad \text{and} \quad \alpha(k) := \frac{kf'(k)}{f(k)}, \tag{7}$$

satisfy the inequality $\sigma(k) \geq 1 - \alpha(k)$ for any $k \in \mathbb{R}_+$, since

$$\sigma(k) = -\frac{1 - \alpha(k)}{\epsilon_{f'}(k)} \quad \text{where} \quad \epsilon_{f'}(k) := \frac{kf''(k)}{f'(k)} > -1.$$

Let $R(c) := -\dfrac{u''(c)}{u'(c)}$ denote the degree of *absolute* risk aversion associated with the utility of the consumer and let

$$\epsilon_R(c) := \frac{cR'(c)}{R(c)}$$

be the elasticity of absolute risk aversion.

Assumption 2 *The utility function u of the consumer is such that the elasticity of absolute risk aversion is between zero and minus one, i. e. $-1 \leq \epsilon_R(c) \leq 0$ for any $c \geq 0$.*

Observe that this assumption stipulates that the degree of absolute risk aversion is a non increasing function with elasticity no less than minus one. In other words, $R(c)$ declines less sharply than $1/c$. Since the degree of *relative* risk aversion is defined as $cR(c)$ the restriction on the elasticity of absolute risk aversion imposes simultaneously that relative risk aversion is non decreasing, since

$$\frac{d}{dc}(cR(c)) = R(c)\big(1 + \epsilon_R(c)\big) \geq 0 \iff -1 \leq \epsilon_R(c).$$

This assumption covers a large class of utility functions, which includes the exponential family $u'(c) = exp\left(-a\frac{c^{1-\gamma}}{1-\gamma}\right)$, containing in particular the isoelastic functions $u(c) = (1/(1-a))\,c^{1-a}$, $a < 1$, as a special case. Moreover, the assumption on the elasticity implies that the absolute risk aversion satisfies $R(y+x) - \delta R(y+\delta x) > 0$, for any $x, y > 0$ and $\delta \in [0,1)$, a useful property needed in the proof of the following proposition[1].

Proposition 1 *If Assumptions 1 and 2 are satisfied, then the inverse demand function $(k,x) \mapsto H(k,x)$ defined in equation (5) is monotonically increasing with respect to its first argument and monotonically decreasing with respect to its second argument[2].*

The above property of the inverse demand function will play an important role in establishing properties of the equilibrium map of capital accumulation under perfect foresight. Market clearing in any period t, with total bond supply $b > 0$ and perfect foresight, implies that next period's capital stock must satisfy the equation

$$k_{t+1} = (1-\tau)w(k_t) - H(k_{t+1}, b)b. \tag{8}$$

Let us first show that (8) yields a unique solution for the level of capital accumulation k_{t+1} for each $k_t \geq 0$ in some compact interval. Let $K > 0$ denote the unique solution of the equation $k = (1-\tau)w(k)$[3]. Then, for any $k_t \in [0, K]$, equation (8) implies that

$$k_{t+1} = (1-\tau)w(k_t) - H(k_{t+1}, b)b \leq (1-\tau)w(k_t) \leq (1-\tau)w(K) = K. \tag{9}$$

Therefore, for any $k_0 < K \Rightarrow k_t < K$ for all $t > 0$. Thus, the interval $[0, K]$ is a forward invariant set of the difference equation (8). Define the function $\Delta : [0, K] \times [0, K] \to \mathbb{R}_+$ as follows

$$(k_{t+1}, k_t) \mapsto \Delta(k_{t+1}, k_t) := k_{t+1} + H(k_{t+1}, b)b - (1-\tau)w(k_t). \tag{10}$$

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[1]see Lemma 1 in the Appendix.

[2]see appendix for the proof

[3]Existence and uniqueness follow from Assumptions 1 and 3 which imply that $w(k)/k$ is strictly decreasing. See also Azariadis & Drazen (1990), pp. 511–512.

It follows from Proposition 1 that Δ is continuous and strictly increasing with respect to its first argument for a given $k_t \in (0, K]$. In addition, $\Delta(0, k_t) = -(1 - \tau)w(k_t) < 0$ and $\Delta(K, k_t) = K + H(K, b)b - (1 - \tau)w(k_t) \geq H(K, b)b > 0$. Applying the Intermediate Value theorem implies the existence and uniqueness of $k_{t+1} \in [0, K]$ solving the equation $\Delta(k_t, k_{t+1}) = 0$. Let $k_{t+1} = G(k_t)$ denote the solution.

Definition 1 *A steady state of the closed economy is a stationary level of capital k^* satisfying $k^* = G(k^*)$.*

It is clear from equation (10), that $k_{t+1} = 0$ solves the equation $\Delta(k_{t+1}, 0) = 0$ implying that $k^* = 0$ is a corner steady state. To guarantee that there exists a unique globally attracting positive steady state one further assumption is needed.

Assumption 3 *For any $k \in \mathbb{R}_+$, the elasticity $\alpha(k)$ of the production function f is less than 0.5, i. e. $\alpha(k) < 0.5$.*

Proposition 2 *If Assumptions 1, 2, and 3 are satisfied, then there exists a unique interior steady state in the closed economy which is globally stable.*

Proof: Define $\phi : [0, K] \to \mathbb{R}_+$ as

$$\phi(k) := [(1 - \tau)w(k) - k]\, r(k) \tag{11}$$

Then, equations (5), (6), and (8) yield that a steady state k must satisfy the equation $\phi(k) = h(k, b)b$. Assumption 3 implies that (i) $\phi(k)$ is strictly decreasing and non-negative for $k \in [0, K]$, and (ii) $\phi(0) = \infty$, and $\phi(K) = 0$. Assumptions 1 and 2 yield that the function $h(k, b)b$ is strictly increasing and non-negative for $k \in [0, K]$. Therefore, existence and uniqueness of an interior steady state follows.

To prove global stability, consider the time one map $G : [0, K] \to [0, K]$. Since the function $(k_{t+1}, k_t) \mapsto \Delta(k_{t+1}, k_t)$ is increasing with respect to its first argument and decreasing with respect to its second argument, it follows from the Implicit Function Theorem that $G(k_t)$ is strictly increasing. Moreover, $G(0) = 0$ and $G(K) < K$. Together with the uniqueness of the interior steady state, this implies that all orbits must be monotonic sequences converging to the interior steady state. Therefore, the steady state is globally stable. □

The role of Assumption 3 is essential in proving existence and uniqueness of an interior steady state. It guarantees that the function $\phi(k)$ is monotonic. It is easy to construct examples where $\phi(k)$ is not monotonic when Assumption 3 is violated. In such cases, there may exist multiple steady states or the economy may have the corner steady state only. Under multiplicity this implies the existence of unstable interior steady states and the occurrence of a poverty trap. While there seems to be no particular theoretical

justification to restrict the elasticity of production to less than one half, as is done in Assumption 3, its impact on the dynamic characteristics of a closed economy with a bond market is striking. For the comparative analysis of the role of a bond market in open economies and in a world economy, to be carried out below, Assumption 3 will be assumed throughout. Therefore, the causes of multiplicity and instability in open economies cannot be attributed to a non convexity of the wage function, identifying the mechanisms associated with bond market integration as the main source of the diverging results.

4   A Two Country World Economy

Next consider a world economy composed of two identical economies of the above type, denoted h (for home country) and f (for foreign country). As before it is assumed that factors of production are immobile across countries, but now, the two bond markets are opened up internationally. In each country, the government issues the same number of bonds b each period while the occurrence of default and the default recovery ratio remains the same. Thus, the return on the two bonds are two random variables which are independent and identically distributed.

With two bonds now available for investment internationally, the consumer in each country has three investment opportunities. Since the two bonds are stochastically the same random variable, the price of the bonds must be the same in equilibrium. In this case, due to risk aversion, consumers will diversify their portfolio of bonds completely, holding always equal amounts of each bond. Thus, under bond market clearing at a uniform price, any feasible bond holding can be defined by a pair $(x, 2b - x)$ where $0 \leq x \leq 2b$ denotes the number of bonds of a country held by its consumers.

Now, let $\mathcal{X}_t^j = (x_t^j/2, x_t^j/2)$ denote the portfolio a young consumer of country $j = h, f$ purchases at time t, containing 50% of the bonds issued by the home government and 50% are bonds issued by the foreign government. Purchasing a portfolio $\mathcal{X}_t^j = (x_t^j/2, x_t^j/2)$, a young consumer faces the budget constraint $i_t + x_t q_t \leq (1 - \tau)w_t$. This, in turn, implies a second period random consumption

$$
c_{t+1} = \begin{cases} (1-\tau)w_t r_{t+1} + x_t(1 - q_t r_{t+1}) & \text{with probability} \quad p^2 \\[2mm] (1-\tau)w_t r_{t+1} + x_t\left(\dfrac{1+\delta}{2} - q_t r_{t+1}\right) & \text{with probability} \quad 2p(1-p) \\[2mm] (1-\tau)w_t r_{t+1} + x_t(\delta - q_t r_{t+1}) & \text{with probability} \quad (1-p)^2. \end{cases} \tag{12}
$$

This can also be expressed as

$$
c_{t+1} = ((1-\tau)w_t - x_t q_t)\, r_{t+1} + x_t\left(1 - \frac{1-\delta}{2}(d_{t+1}^h + d_{t+1}^f)\right), \tag{13}
$$

where $d_{t+1}^j \in \{0,1\}$ denotes government j's default indicator. Again, taking into consideration the law of capital accumulation in each country $k_{t+1} = (1-\tau)w(k_t) - x_t q_t$, one obtains that the random second period consumption can be written as

$$\bar{\bar{c}}_{t+1} = \rho(k_{t+1}) + x_t, \quad \bar{c}_{t+1} = \rho(k_{t+1}) + \frac{1+\delta}{2}x_t, \quad \text{and} \quad \underline{\underline{c}}_{t+1} = \rho(k_{t+1}) + \delta x_t, \qquad (14)$$

where $\bar{\bar{c}}_{t+1}$, $\bar{c}_{t+1}$ and $\underline{\underline{c}}_{t+1}$ are second period consumptions, when neither of the governments defaults on its obligation, one of the governments default, and both governments default on their obligations respectively. Using this notation, one can write the consumer's optimization problem as

$$\max_{x_t \geq 0} \left\{ p^2 u(\bar{\bar{c}}_{t+1}) + 2p(1-p)u(\bar{c}_{t+1}) + (1-p)^2 u(\underline{\underline{c}}_{t+1}) \mid q_t x_t \leq (1-\tau)w_t \right\}, \qquad (15)$$

and one obtains as the first order condition for an interior solution

$$p^2 u'(\bar{\bar{c}}_{t+1})\left(1 - q_t r_{t+1}\right) + 2p(1-p)u'(\bar{c}_{t+1})\left(\frac{1+\delta}{2} - q_t r_{t+1}\right) + (1-p)^2 u'(\underline{\underline{c}}_{t+1})\left(\delta - q_t r_{t+1}\right) = 0. \qquad (16)$$

This implies again an explicit inverse demand function for bonds

$$q_t = \Pi(k_{t+1}, x_t) := \frac{\pi(k_{t+1}, x_t)}{r(k_{t+1})}, \qquad (17)$$

where

$$\pi(k_{t+1}, x_t) := \frac{p^2 u'(\bar{\bar{c}}_{t+1}) + p(1-p)(1+\delta)u'(\bar{c}_{t+1}) + (1-p)^2 \delta u'(\underline{\underline{c}}_{t+1})}{p^2 u'(\bar{\bar{c}}_{t+1}) + 2p(1-p)u'(\bar{c}_{t+1}) + (1-p)^2 u'(\underline{\underline{c}}_{t+1})}, \qquad (18)$$

is the risk adjusted second period payment of the portfolio $\mathcal{X}_t = (x_t/2, x_t/2)$.

Proposition 3 *If Assumptions 1 and 2 are satisfied, then the inverse demand function* $(k, x) \mapsto \Pi(k, x)$ *defined in equations (17) and (18) is monotonically increasing with respect to its first argument and monotonically decreasing with respect to its second argument*[4].

The properties of the inverse demand function can be used to define the direct portfolio demand function. Suppose the pair (k_{t+1}^h, k_{t+1}^f) is such that the following inequalities are satisfied $\Pi(k_{t+1}^h, 0) > \Pi(k_{t+1}^f, 2b)$ and $\Pi(k_{t+1}^h, 2b) < \Pi(k_{t+1}^f, 0)$. Then, the monotonicity property of the function Π implies that the equation

$$\Pi(k_{t+1}^h, x) = \Pi(k_{t+1}^f, 2b - x). \qquad (19)$$

has a unique solution with respect to x. Let $X(k_{t+1}^h, k_{t+1}^f)$ denote this solution. We set $X(k_{t+1}^h, k_{t+1}^f) = 0$ when $\Pi(k_{t+1}^h, 0) \leq \Pi(k_{t+1}^f, 2b)$ and set $X(k_{t+1}^h, k_{t+1}^f) = 2b$ when

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[4]The proof of this proposition is similar to the one of Proposition 1.

$\Pi(k_{t+1}^h, 2b) \geq \Pi(k_{t+1}^f, 0)$. The function $X : [0, K] \times [0, K] \to [0, 2b]$ denotes the demand of an equally weighted portfolio chosen by a consumer of the home country. By definition, the following identity $X(k_{t+1}^h, k_{t+1}^f) + X(k_{t+1}^f, k_{t+1}^h) \equiv 2b$ is satisfied for any pair $(k_{t+1}^h, k_{t+1}^f) \in [0, K] \times [0, K]$.

The direct portfolio demand function can now be used to define the dynamics under perfect foresight. For a given pair $(k_t^h, k_t^f))$, next period's capital stock in the home and in the foreign country (k_{t+1}^h, k_{t+1}^f) should satisfy the system of equations

$$\begin{cases} k_{t+1}^h = (1-\tau)w(k_t^h) - X(k_{t+1}^h, k_{t+1}^f)\Pi\left(k_{t+1}^h, X(k_{t+1}^h, k_{t+1}^f)\right) \\ k_{t+1}^f = (1-\tau)w(k_t^f) - X(k_{t+1}^f, k_{t+1}^h)\Pi\left(k_{t+1}^f, X(k_{t+1}^f, k_{t+1}^h)\right). \end{cases} \tag{20}$$

Let $k_{t+1}^h = \Phi(k_t^h, k_t^f)$ and $k_{t+1}^f = \Phi(k_t^f, k_t^h)$ denote the solution of the above system. Observe that the two equations are perfectly symmetric to each other, namely they are obtained by a simple permutation of the respective arguments for the inverse demand function Π and for the direct demand function X, implying that the dynamical system is symmetric.

Definition 2 *A steady state in the world economy is a stationary pair of capital (k^h, k^f) such that $k^h = \Phi(k^h, k^f)$ and $k^f = \Phi(k^f, k^h)$.*

Since the world economy is closed as a whole, it follows from the analysis of a closed economy that there are two symmetric steady states in the world economy $(0, 0)$ and $(k^\star, k^\star)$ where $k^\star$ solves the equation

$$[(1-\tau)w(k) - k]\, r(k) = \pi(k, x)\, x. \tag{21}$$

with $x = b$. In both steady states agents of both countries hold a portfolios $\mathcal{X} = (b/2, b/2)$ containing a total of b units of bonds.

The implicit form (20) of the dynamical system (Φ, Φ) shows, that the local stability of the symmetric steady state $(k^\star, k^\star)$ will depend in a complex manner on the partial derivatives of the indirect and the direct demand functions Π and X. These describe the two sided symmetric spill over effects which are at the center of the interaction between the two economies when their consumers participate jointly in the two fully symmetric markets. These spill over effects drive the symmetry breaking result below. They are responsible for the destabilizing forces which occur when two autarkic economies combine their bond markets.

In view of the next proposition, it is useful to introduce the following notation. Let

$$\epsilon_k(k, x) = \frac{k\Pi_k(k, x)}{\Pi(k, x)} \quad \text{and} \quad \epsilon_x(k, x) = \frac{x\Pi_x(k, x)}{\Pi(k, x)} \tag{22}$$

denote the elasticities of the inverse demand function with respect to next period's capital and to bond holdings and define their values at the steady state as

$$\epsilon_k^\star = \epsilon_k(k^\star, b) \quad \text{and} \quad \epsilon_x^\star = \epsilon_x(k^\star, b). \tag{23}$$

Similarly, let $q^\star = \Pi(k^\star, b)$ denote the bond price at the symmetric steady state, $\alpha^\star$ the capital share in production, and $\sigma^\star$ the elasticity of substitution.

Now, define a critical value $\mu^\star$ as

$$\mu^\star := \epsilon_k^\star + \epsilon_x^\star \left(1 + \frac{1}{s^\star} \left(\frac{\alpha^\star}{\sigma^\star} - 1 \right) \right), \tag{24}$$

where

$$s^\star = \frac{bq^\star}{(1-\tau)w(k^\star)} \quad \text{and} \quad 1 - s^\star = \frac{k^\star}{(1-\tau)w(k^\star)} \tag{25}$$

denote the proportion of net wage income spent on the bond market and on physical capital investment respectively.

It is clear that, after combining the two bond markets, there are two asymmetric steady states, in which one country buys all bonds and holds a positive level of capital and the other deteriorates to zero level capital and income. Thus, $(0, \widetilde{k})$ and $(\widetilde{k}, 0)$ are two asymmetric steady states, where $\widetilde{k}$ solves the equation (21) with $x = 2b$. In order to answer whether there exist *asymmetric and interior* steady states in which both countries hold a positive capital stock, we introduce the following notation. Let $k = \xi(x)$ denotes the unique solution of equation (21) with respect to k for any given $x \in [0, 2b]$. Let the function $\Lambda : [0, 2b] \to \mathbb{R}$ be defined as follows

$$\Lambda(x) := \Pi(\xi(x), x) - \Pi(\xi(2b - x), 2b - x), \tag{26}$$

which is the difference of the supporting stationary bond prices of the two economies. This must be zero at any steady state of the world economy. Proposition (3) implies that $\Lambda(0) > 0$, $\Lambda(2b) < 0$, and that equalization of bond prices implies that $\Lambda(b) = 0$. Therefore, for the existence of at least two asymmetric steady states it is enough to show that $\Lambda'(b) > 0$.

Proposition 4 *Let Assumptions 1, 2, and 3 be satisfied. If $\mu^* < 0$ then $\Lambda'(b) > 0$ and there exist at least two interior asymmetric steady states.*

Figure 1 illustrates two cases with multiple interior asymmetric steady states. Panel (b) reveals that the condition of the proposition is not necessary.

Given the symmetry of the dynamic system (20), it is known that there can be no complex eigenvalues. However, the occurrence of interior asymmetric steady states is intimately related to the instability of the symmetric steady state. This insight leads us to the main

result, which consists of identifying values for the government parameters (b, p) which cause symmetry breaking. Let $\Omega := \mathbb{R}_+ \times (0, 1)$ denote the space of the government parameters and let

$$\Omega^u = \{(b, p) \in \Omega | \mu^\star < 0\}, \quad \Omega^c = \{(b, p) \in \Omega | \mu^\star = 0\}, \quad \text{and} \quad \Omega^s = \{(b, p) \in \Omega | \mu^\star > 0\} \tag{27}$$

be a partition of Ω.

Proposition 5 *Let the Assumptions 1, 2, and 3 are satisfied.*

- *The interior steady state (k^*, k^*) has two positive real roots.*

- *(k^*, k^*) is asymptotically stable only if $(b, p) \in \Omega^s$.*

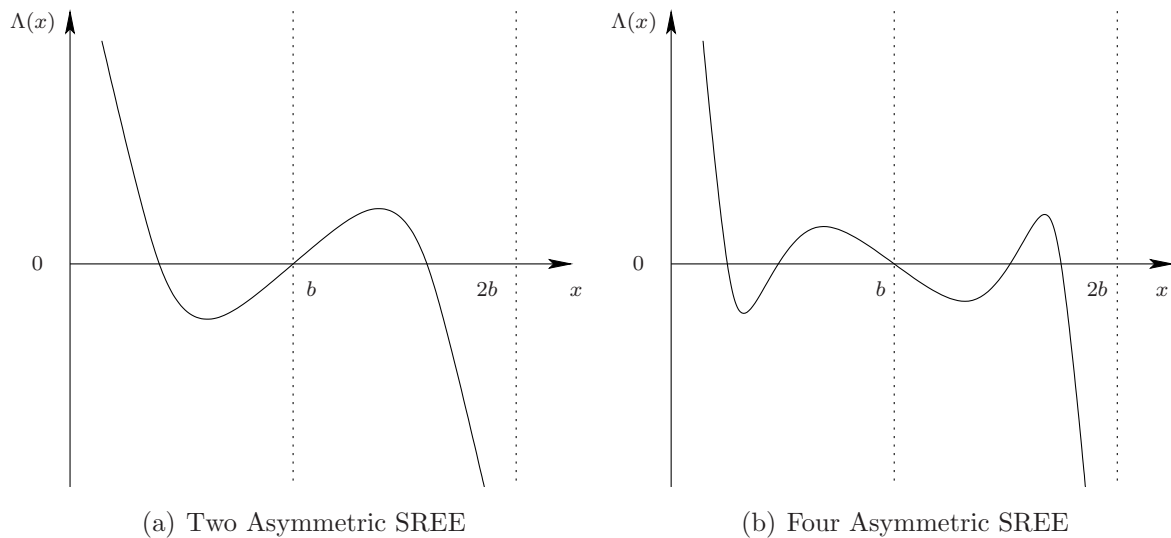- *(k^*, k^*) becomes unstable via a fold bifurcation, as (b, p) leaves the region Ω^s.*

(a) Two Asymmetric SREE   (b) Four Asymmetric SREE

Figure 1: Existence of Asymmetric Steady States

From Proposition 3 one knows that $\epsilon_k^\star > 0 > \epsilon_x^\star$. The first term in equation (24) corresponds essentially to the positive effect on real capital accumulation in each country inducing the stabilizing force of domestic capital formation, which corresponds to the stabilizing effect under autarky. The second effect, however, is an induced crowding out effect on capital formation of additional bond holding. As (24) shows, its size depends on the relative magnitudes of several parameters of the model induced by the production function and by the risk aversion of consumers. If the second effect dominates the first effect in size, the critical level reverses its sign and instability occurs. The proof of the proposition (see appendix) reveals that the condition on the critical level of μ for local instability is the same as the one in the proposition for multiple asymmetric steady states.

Thus, instability implies the occurrence of multiple interior steady states. Therefore, stable asymmetric steady states exist, whenever symmetry breaking occurs. However, there may exist multiple interior asymmetric steady states even when the symmetric state is stable (see Figure 1(b)).

A Numerical Example:

In order to demonstrate the possibility of endogenously determined inequality and the instability of the symmetric steady state we analyze a parameterized version of a world economy of the above type. Suppose that the production function is Cobb-Douglas $f(k) = Ak^\alpha$, and that the utility function is of the exponential family such that its derivative has the form

$$u'(c) = \exp\left(-a\frac{c^{1-\gamma}}{1-\gamma}\right) \qquad a > 0, \qquad \gamma \geq 0.$$

The absolute risk aversion is $R(c) = ac^{-\gamma}$ with elasticity $-\gamma$. It is clear that the production function satisfies Assumptions 1 and 3 when $\alpha < 0.5$. The utility function satisfies Assumption 2 when $a > 0$ and $\gamma \in [0, 1]$. The absolute risk aversion is constant with $R(c) = a$ when $\gamma = 0$, while it becomes $R(c) = ac^{-1}$ when $\gamma = 1$. Thus, the above specification includes the two utility functions CARA and CRRA as special cases.

In order to determine parameter regions under which the unique symmetric steady state is unstable, we first determine a list of parameters (given in Table 1) for which instability

A	α	τ	b	p	δ	a	γ
1.00	0.40	0.30	4.00	0.95	0.15	1.40	0.09

Table 1: Standard parameter set

occurs. Then, we perform pairwise deviations from this reference set and determine numerically the range for which the instability is preserved. Clearly, continuity of the

(a) b - a parameter region   (b) b - γ parameter region   (c) p - γ parameter region
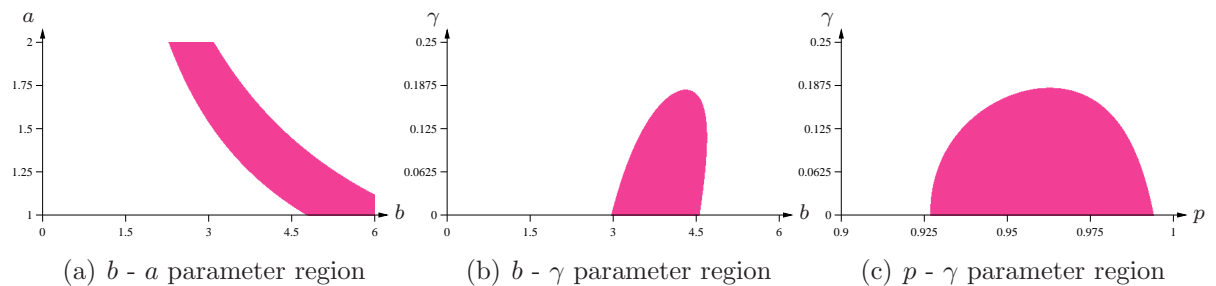
Figure 2: Parameter Regions with Unstable Symmetric Steady State

steady state (k^*, k^*) with all parameters of the system implies that the instability is preserved on open set. In other words drawing $(a, b, p, \delta, \gamma) \in \mathbb{R}^5_+$ locally will display a section of Ω^u for which (k^*, k^*) is unstable. Figure 2 displays sections of the parameter

region Ω^u of instability in red/dark. Thus, in these regions there exist stable interior asymmetric steady states. Conversely, the world economy has an interior symmetric steady state only when the parameter values belong to the white region. Therefore, there exists an open set of the parameters $(a, b, p, \delta, \gamma) \in \mathbb{R}_+^5$ for which symmetry breaking occurs.

5 Summary and Conclusions

The paper provides a theoretical explanation of the occurrence of inter-country income inequality and of the co-existence of rich and poor countries in the long run. A fully neoclassical growth model with two identical economies is considered where consumers hold portfolios of real capital and risky government discount bonds. Consumers have rational expectations and all markets operate under perfect competition. One of the basic features in such economies is given by the fact that government bonds have a crowding out effect on real capital, in spite of the fact that consumers regard the acquisition of a discount bond as a form of personal savings and investment. This is because, in the portfolio decision of consumers, the income spent on the bond market reduces investment in physical capital which results in the trading of bonds for consumption purposes. Thus, government bonds crowd out real investment.

In a world economy with two identical countries their capital accumulation converges to a unique and globally symmetric steady state when each bond market operates only domestically. This symmetric world equilibrium remains as a steady state when the two bond markets are joined. However, if the bond markets open internationally, the equalization of the bond price may generate instability in each country's capital accumulation near the symmetric steady state. This is caused by an interaction of the elasticities of substitution in production and of bond demand which appears only for each economy when it becomes an open economy and its consumers are able to diversify their risks between the internationally available bonds of two governments. Therefore, when two such economies internationalize their bond markets *prior* to convergence under autarky, the dynamics in the world economy does not lead to the symmetric steady state even for slightly asymmetric initial capital endowments. In such cases the distribution of capital at the time of internationalization becomes the major determinant of the differential features of steady states in a world with otherwise identical economies. When the uncertainty of the government default is removed and the bonds become perfect substitutes for capital in the portfolio decision of consumers, the instability no longer occurs. Thus, the existence of sovereign risk *and* the internationalization of the associated bond markets are the cause for symmetry breaking among identical or similar nations.

6 Appendix

Lemma 1 *If Assumption 2 is satisfied then $R(y+x) - \delta R(y+\delta x) > 0$, for any $x, y > 0$ and $\delta \in [0, 1)$.*

Proof: Let us fix values $x, y > 0$ and define the function $\mathcal{R} : [0, 1] \to \mathbb{R}$ as follows $\delta \mapsto \mathcal{R}(\delta) \equiv R(y+x) - \delta R(y+\delta x)$. From this it follows that

$$\mathcal{R}'(\delta) = -R(y+\delta x) - \delta x R'(y+\delta x) = -R(y+\delta x)\left[1 - \frac{\delta x}{y+\delta x}\epsilon_R(c)\right] \leq 0, \qquad (28)$$

because, $x, y > 0$, the function R is non-negative and $\epsilon_R \in [0, 1]$. Since $\mathcal{R}(0) = R(y+x) > 0$ and $\mathcal{R}(1) = 0$ the claim of the lemma follows.

$\square$

Lemma 2 *If $\alpha(k) < 0.5$, then the function $k \mapsto \phi(k)$ defined in (11) satisfies: $\phi(0) = \infty$, $\phi(\overline{k}) = 0$, and ϕ is a strictly decreasing and non-negative.*

Proof: By definition $\phi(k) = (1-\tau)w(k)r(k) - kr(k)$. When $1 - \sigma(k) \leq \alpha(k) < 0.5$, then the function ϕ is decreasing because, a)when $\alpha(k) < 0.5$ then $[w(k)r(k)]' = w'(k)r(k) + w(k)r'(k) = r'(k)\left(w(k) - kr(k)\right) < 0$ and b) when $1 - \sigma(k) \leq \alpha(k)$ then capital income $kr(k)$ is an increasing function of capital. $\phi(K) = 0$ by definition of K. In addition $\lim_{k \to 0} \phi(k) = (1-\tau)\lim_{k \to 0} w(k)r(k) = (1-\tau)\lim_{k \to 0} k^{2\alpha(k)-1} = \infty$. For $(1-\tau)w(k) > k$ for $k \in [0, K]$ and thus ϕ is non negative.

$\square$

Proof of Proposition 1:

Let us define a function $\psi : \mathbb{R}_+^2 \to \mathbb{R}$ as follows

$$(k, x) \mapsto \psi(k, x) = \frac{1-p}{p}\frac{u'(\rho(k)+\delta x)}{u'(\rho(k)+x)}. \qquad (29)$$

Then

$$h(k, x) = \frac{1 + \delta\psi(k, x)}{1 + \psi(k, x)}. \qquad (30)$$

By taking a natural log of both sides of equation (30) and then differentiating it we obtain

$$\frac{kh_k(k, x)}{h(k, x)} = \frac{(1-\delta)k\psi(k, x)}{(1 + \delta\psi(k, x))(1 + \psi(k, x))}\left[R(\rho(k)+\delta x) - R(\rho(k)+x)\right]\rho'(k) \qquad (31)$$

and

$$\frac{xh_x(k, x)}{h(k, x)} = \frac{(1-\delta)x\psi(k, x)}{(1 + \delta\psi(k, x))(1 + \psi(k, x))}\left[\delta R(\rho(k)+\delta x) - R(\rho(k)+x)\right]. \qquad (32)$$

When Assumption 1 is satisfied then the function $\rho(k) = kr(k)$ is increasing. When Assumption 2 is satisfied then agent's absolute risk aversion function is decreasing and thus the following inequality holds

$$\frac{kH_k(k,x)}{H(k,x)} = \frac{kh_k(k,x)}{h(k,x)} - \frac{kr'(k)}{r(k)} > 0. \tag{33}$$

As a result, the function $(k,x) \mapsto H(k,x)$ is increasing with respect to its first argument.

When Assumption 2 is satisfied then the Lemma 1 implies that $R(\rho(k)+x) - \delta R(\rho(k) + \delta x) > 0$ for any $\delta \in [0,1)$. This with equation (32) implies that

$$\frac{xH_x(k,x)}{H(k,x)} = \frac{xh_x(k,x)}{h(k,x)} < 0. \tag{34}$$

As a result, the function $(k,x) \mapsto H(k,x)$ is decreasing with respect to its second argument.

$\square$

Proof of Propositions 4 and 5:

$$\mu^\star = \epsilon_k^\star + \epsilon_x^\star \left(1 + \frac{1}{s^\star}\left(\frac{\alpha^\star}{\sigma^\star} - 1\right)\right) < 0 \quad \Leftrightarrow \quad \frac{\alpha^\star}{\sigma^\star} > 1 - s^\star - s^\star\frac{\epsilon_k^\star}{\epsilon_x^\star}. \tag{35}$$

(35) can be rewritten

$$\frac{k^\star w'(k^\star)}{w(k^\star)} > \frac{k^\star}{(1-\tau)w(k^\star)} - \frac{bq^\star}{(1-\tau)w(k^\star)}\frac{k^\star\Pi_k^\star}{b\Pi_x^\star}. \tag{36}$$

(36) implies that

$$(1-\tau)w'(k^\star) - 1 > -\frac{\Pi_k^\star}{\Pi_x^\star}\Pi^\star \quad \Leftrightarrow \quad \left((1-\tau)w'(k^\star) - 1\right)\Pi_x^\star + \Pi_k^\star\Pi^\star < 0. \tag{37}$$

Since $(1-\tau)w'(k^\star) - 1 - \Pi_k^\star b < 0$ it follows from (37) and from equations

$$\Lambda'(b) = \Pi_k^\star\xi'(b) + \Pi_x^\star \quad \text{and} \quad \xi'(b) = \frac{\Pi_x^\star b + \Pi^\star}{(1-\tau)w'(k^\star) - 1 - \Pi_k^\star b} \tag{38}$$

that $\Lambda'(b) > 0$ if and only if (35) is satisfied.

$\square$

References

AZARIADIS, C. & A. DRAZEN (1990): "Threshold Externalities in Economic Development", *Quarterly Journal of Economics*, 105, 501–526.

BÖHM, V., T. KIKUCHI & G. VACHADZE (2007): "Risk, Factor Substitution, and Asset Market Integration", in *Dynamik Internationaler Märkte, Wirtschaftswissenschaftliches Seminar Ottobeuren*, ed. by W. Franz, J. Ramser, H. & M. Stadler, vol. 36, pp. 49–69. Mohr Siebeck, Tübingen.

BÖHM, V. & G. VACHADZE (2009): "Endogeneous Inequality of Nations Through Financial Asset Market Integration", *Macroeconomic Dynamics*, In Press.

BOYD, J. H. & B. D. SMITH (1997): "Capital Market Imperfections, International Credit Markets, and Nonconvergence", *Journal of Economic Theory*, 73, 335 – 364.

FISCHER, S. (1998): "Capital Account Liberalization and the Role of the IMF", *Princeton Essays in International Finance*, 207, 1–10.

KIKUCHI, T. (2008): "International Asset Market, Nonconvergence, and Endogenous Fluctuations", *Journal of Economic Theory*, 139, 310–334.

KIKUCHI, T. & J. STACHURSKI (2009): "Endogenous Inequality and Fluctuations in a Two-country Model", *Journal of Economic Theory*, In Press.

KRUGMAN, P. R. & A. J. VENABLES (1995): "Globalization and the Inequality of Nations", *Quarterly Journal of Economics*, 110, 857–880.

MATSUYAMA, K. (1996): "Why Are There Rich and Poor Countries?: Symmetry-Breaking in the World Economy", *Journal of the Japanese and International Economies*, 10, 419–439.

MATSUYAMA, K. (2004): "Financial Market Globalization, Symmetry–Breaking, and Endogenous Inequality of Nations", *Econometrica*, 72(3), 853–884.

MATSUYAMA, K. (2006): "Symmetry-Breaking", *New Palgrave: A Dictionary of Economics*, 2-nd edition, Macmillan.

SUMMERS, L. H. (2000): "International Financial Crises: Causes, Prevention, and Cures", *American Economic Review*, 90, 1-16.